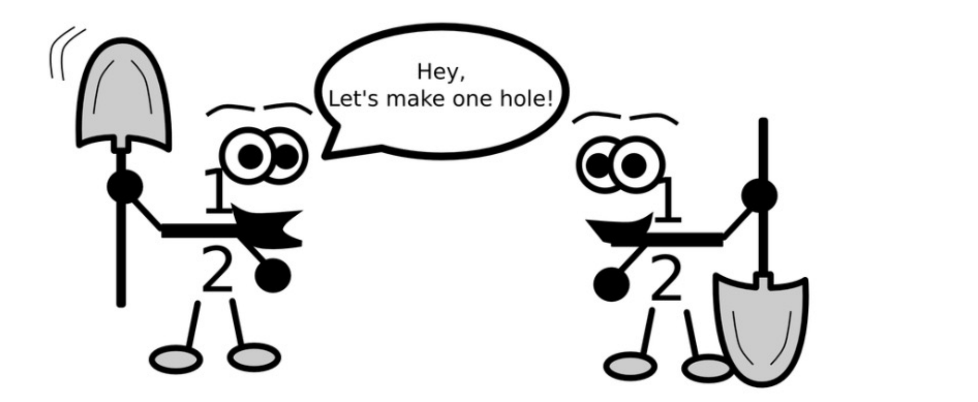
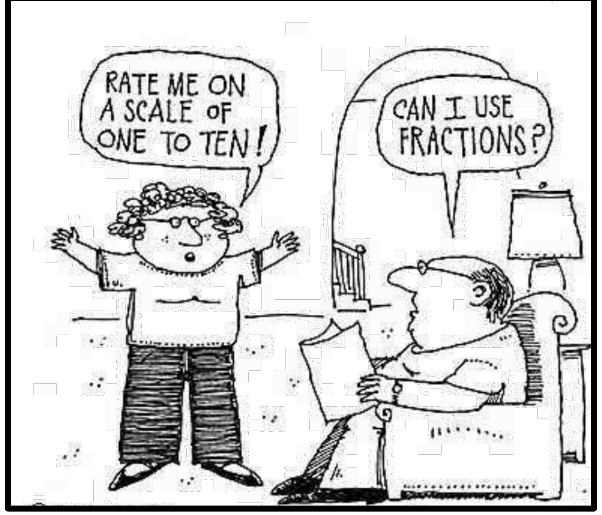
"I'd like to meet you halfway, but I'm terrible with fractions."





Fraction Basics

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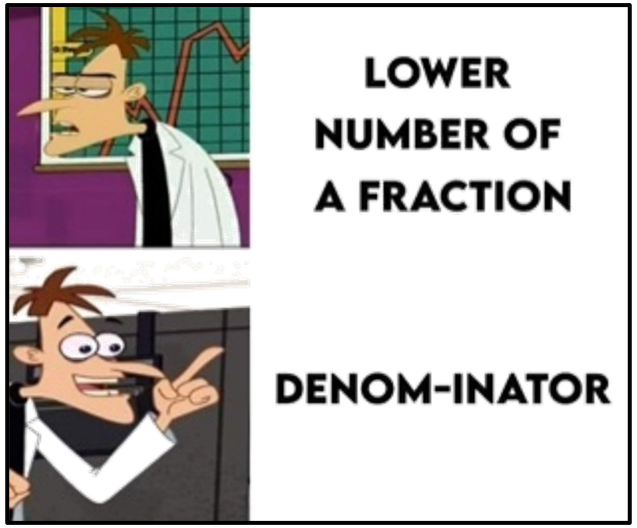
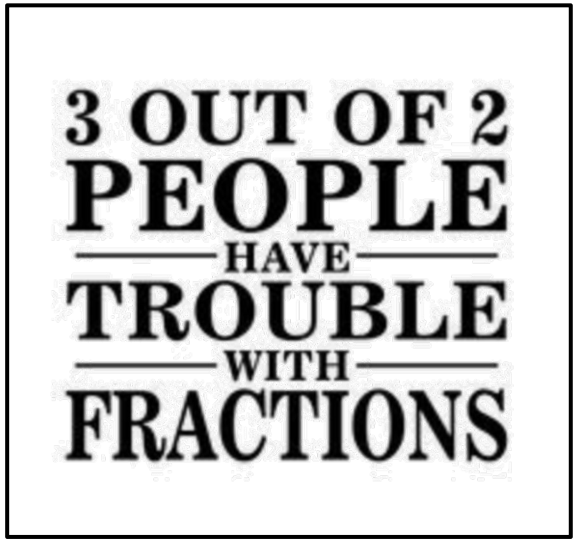
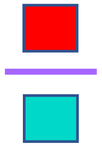
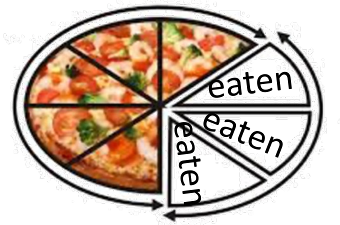


What Is A Fraction?



There are two parts of a fraction, the top number and the bottom number 

The number on top is called the **numerator** and tells how many parts of the whole there is. The bottom number is called the **denominator** and tells how many parts of a fraction make a whole (the number of parts a whole is divided into).

Consider a whole pizza  Let's say we cut the pizza into 8 equal pieces and someone eats 3 pieces 

There are 5 parts left and 8 parts in total (number of parts the whole is divided into) which is written as $\frac{5}{8}$

So, a fraction is a number that is used to **represent a whole number that has been divided into equal parts**.

A fraction represents part of something. We can also say there are 5 slices (5 parts) out of a total of 8 slices (the whole).

$\frac{5}{8}$

5 is called the numerator

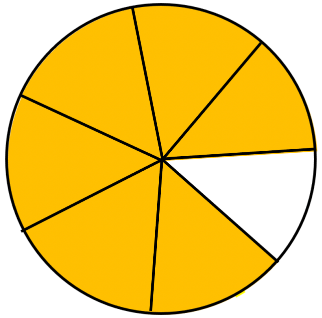
8 is called the denominator

Why are fractions used? Fractions are important because they tell you the portion of a whole. So, a fraction talks about how many parts of a whole (8 parts in that whole) there are.

Which of the following fractions are smaller than $\frac{6}{7}$?

$\frac{3}{5}$ $\frac{5}{6}$ $\frac{3}{4}$ $\frac{6}{8}$ $1\frac{1}{5}$

Let's represent $\frac{6}{7}$ first

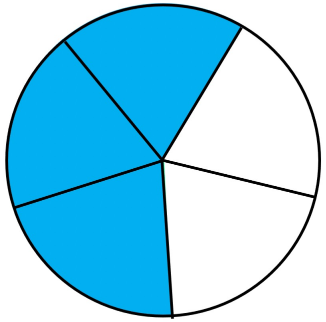


$\frac{6}{7}$

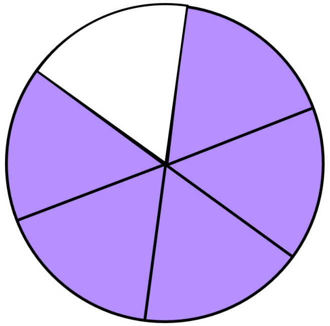
Now let's represent all fractions

Notice how these are the same

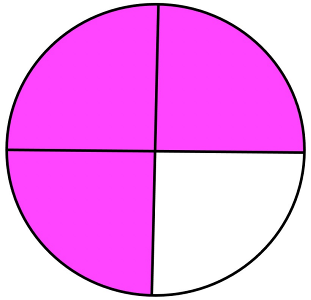
$\frac{3}{5}$



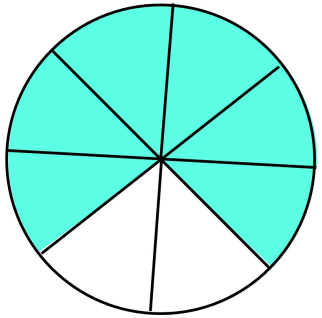
$\frac{5}{6}$



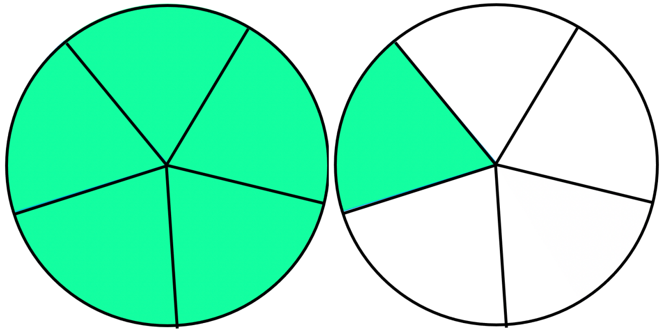
$\frac{3}{4}$



$\frac{6}{8}$

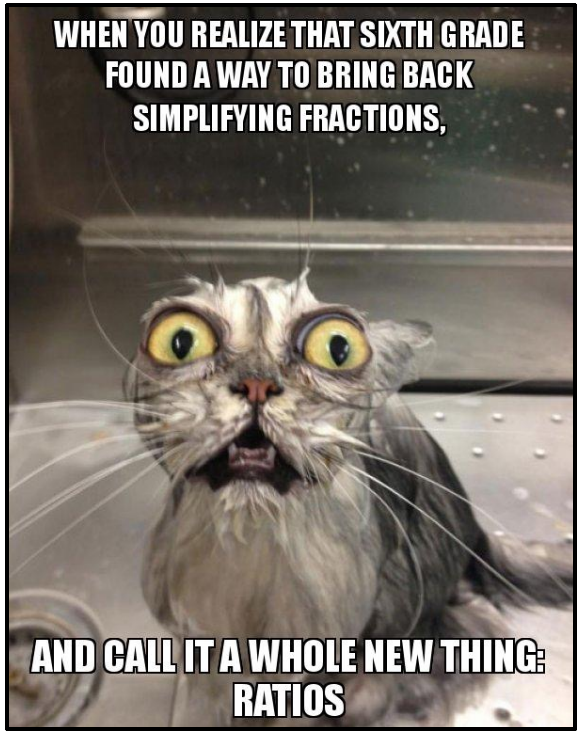


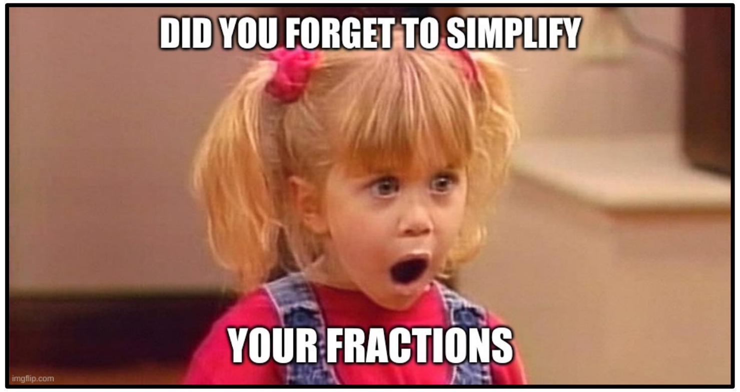
$1\frac{1}{5}$.



$\frac{3}{5}$ $\frac{5}{6}$ $\frac{3}{4}$ and $\frac{6}{8}$ are all smaller than $\frac{6}{7}$

Simplifying Fractions





Basic Method

We check whether the following numbers fit:

2 ,3 5, 7 and 11

We can remember these numbers by ”at 2:35 we go 7-11”

It is useful to remember that:

Divisible by 2: ends in an even number

Divisible by 3: sum of digits is divisible by 3

Divisible by 5: ends in 0 or 5

Example 1: $\frac{15}{18}$

Try 2, 3, 5, 7, 11 in order

2 doesn’t work

3 works

$$= \frac{3\times5}{3\times6}$$

Cancel the common letters

$$= \frac{\cancel{3}\times5}{\cancel{3}\times6} = \frac{5}{6}$$

Example 2: $\frac{16}{20}$

Try 2, 3, 5, 7, 11 in order

2 works

$$= \frac{\cancel{2}\times8}{\cancel{2}\times10} = \frac{8}{10}$$

2 works again

$$\frac{8}{10} = \frac{\cancel{2}\times4}{\cancel{2}\times5} = \frac{4}{5}$$

Using the biggest factor would have gotten us our answer quicker. For example, if we chose 4 instead:

$$\frac{16}{20} = \frac{4\times4}{4\times5} = \frac{4}{5}$$

For weak students: Check if both numbers are even. If they are, take half and keep taking half until you can’t anymore. This is helpful for students who don’t know their multiplication facts, but doesn’t always work as the numbers aren’t always even

If you want to be super quick, learn all your divisibility rules:

Divisibly by 2: ends in an even number
Divisible by 3: sum of digits is divisible by 3
Divisibly by 4: Last 2 digits divisible by 4
Divisible by 5: ends in 0 or 5
Divisibly by 6: check divisible by 2 and 3
Divisible by 8: Last 3 digits divisible by 8
Divisible by 9: sum of digits is divisible by 9
Divisible by 10: ends in 0

Ladder/Cake Method
(Great method!)

Simplify $\frac{12}{36}$

1. Write the numbers: Write the numerator and the denominator side by side   12   36

2. Draw a cake layer: Draw an “L” or a cake layer underneath the numbers.

“L”

3 | 12   36

“Cake Layers”

| 12   36 |

3. Find a Common Factor: Do a little detective work and find a number that both numbers can be divided by. The great thing about this method is that *any* factor will do. Students don’t have to worry about finding the largest common factor. Let’s choose 3 since we can divide 12 and 36 by 3.

4. Write the Factor: Write the chosen factor to the left of the “L” or cake layer.

“L”

3 | 12   36

“Cake Layers”

3 | 12   36 |

5. Divide: Divide both numbers by the factor and write the result (quotients) underneath.

“L”

3 | 12   36

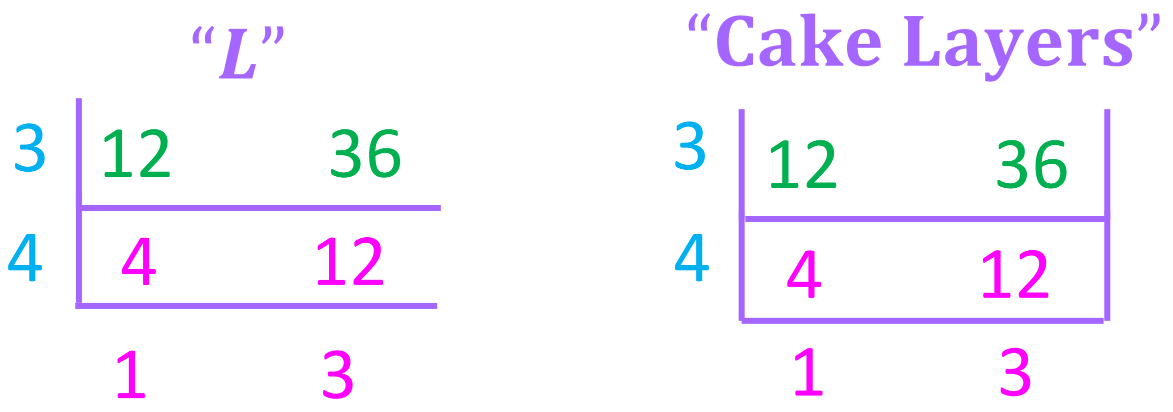
4   12

“Cake Layers”

3 | 12   36 |

4   12

6. Repeat: Draw another "L" or cake layer and repeat the process until there is no number that both numbers can be divided by. We can divide by 4.



1 and 3 are prime, we are done

7. Voilà, Your Simplified Fraction!: The remaining numbers at the bottom are the numerator and denominator of your simplified fraction.

$$\frac{1}{3}$$

Hence

$$\frac{12}{36} \text{ can be simplified to } \frac{1}{3}$$

Students' work may look different when using the ladder method/cake method). For instance, when simplifying $\frac{36}{48}$, one student may recognize that the numerator and denominator are divisible by 2, while another student may start by dividing both by 3 or even start by dividing by 12.

Way 1: Divide by 2 first

2	36	48
2	18	24
3	9	12
	3	4

There is no number that fits in 3 and 4 and we are done

Way 2: Divide by 2 first

3	36	48
4	12	16
	3	4

There is no number that fits in 3 and 4 and we are done

Way 3: Divide by 12 first

12	36	48
	3	4

There is no number that fits in 3 and 4 and we are done

All give us the same answer which is $\frac{3}{4}$

Hence

$$\frac{36}{48} \text{ can be simplified to } \frac{3}{4}$$

This is not part of simplifying fractions, but the ladder/cake method is very useful for other topics such as finding the **Highest Common Factor** HCF (aka Greatest Common Factor GCF) , finding the **lowest common multiple** LCM and also **prime factorisation**

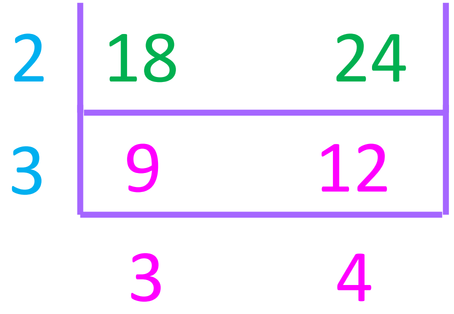
FINDING THE HCF: WITH THE LADDER METHOD:

All you have to do is look at the numbers to the left of the Cake once all the layers have been completed. Multiply these numbers together to get the GCF.

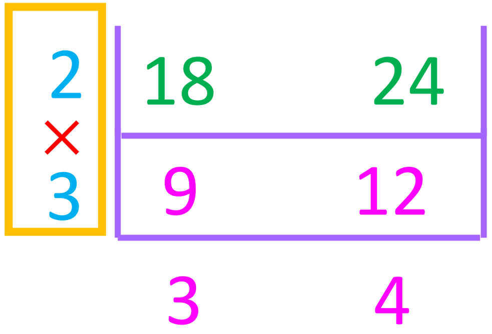
FINDING THE LCM:

Students can find the least common multiple of the two numbers by drawing an “L” around the numbers on the outside of the cake. Then, multiply all these numbers together to get the LCM.

Example: Let’s find the HCF and LCM of 18 and 24. We first apply the cake/ladder method.

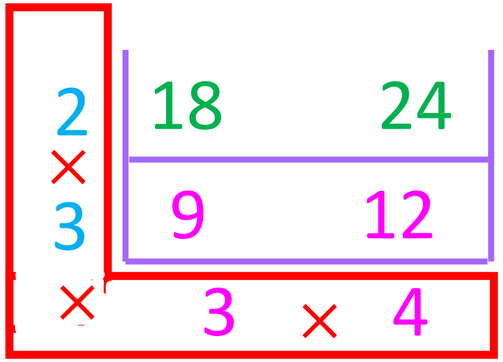


Method for HCF:



HCF= $2 \times 3 = 6$

Method for LCM:

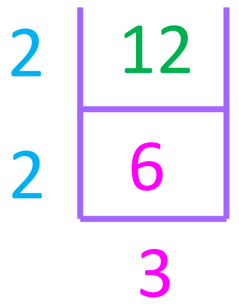


LCM= $2 \times 3 \times 3 \times 4 = 72$

FINDING PRIME FACTORISATION:

Example 1:

Find the prime factorisation of 12



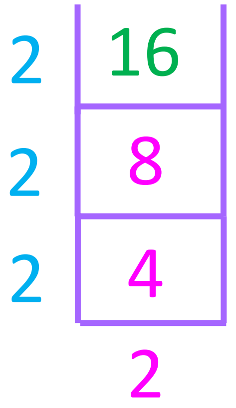
The numbers outside form the product

$$2 \times 2 \times 3$$

$$= 2^2 \times 3$$

Example 2:

Find the prime factorisation of 16



The numbers outside form the product

$$2 \times 2 \times 2 \times 2$$

$$= 2^4$$

More Advanced Method - Factors and Highest Common Factors

Let's first define clearly what it means to "**simplify**" in terms of fractions:

Simplifying a fraction is **finding a common factor** and dividing both the numerator and denominator by it (more simply put - **finding the number that fits** into both the numerator and denominator and **saying how many times** for each).

Finding not just any common factor, but the **highest** common factor instead of just any common factor means that you can simplify in a shorter number of steps!

So, what is this **number that fits in** and what is a **common factor** or **highest common factor**? First of all, let's define a factor. Factors are numbers that we can multiply together to get another number. A number can have several factors.

For example, consider the numbers 18 and 12.

18 has the factors 1,2,3,6,9,18
12 has the factors 1,2,3,4,6,12

Note: To make sure that you never miss any factors you can always write them out in pairs

For **18** we can write 1 and 18, 2 and 9, 3 and 6 which then gives us the list of factors 1,2,3,6,9,18
For **12** we can write 1 and 12, 2 and 6, 3 and 4 which then gives us the list of factors 1,2,3,4,6,12

The **common factors** are those that are found in both lists.

Factors of 18: 1,2,3,6,9,18
Factors of 12: 1,2,3,4,6,12

Therefore,

- The common factors of 12 and 18 are 1,2,3 and 6
- The highest common factor is the greatest of the common factors which is 6
- 6 fits into 12 two times

Why is knowing the greatest/highest common factor useful with fractions?

We put a fraction into its lowest terms by finding a common factor (recall that this is a whole number that goes into both the numerator and denominator with no remainder) and then dividing by it. Repeat this process until there is no longer a number that goes into both the numerator and denominator.

Step By Step Method Of Simplifying:

Step 1: Find a common factor of both numbers i.e. a number that fits into both the numerator AND denominator

Step 2: Say how many times for each

Step 3: Check you can't do steps 1 and 2 again. If you can't then you're done!

Note: If you chose the HIGHEST common factor step 1, then you wouldn't have to do step 3.

Let's look at an example: Simplify $\frac{18}{27}$

Pick any common factor

$\frac{18}{27}$

Find ANY number (ANY common factor) that **fits** into both the numerator and denominator and say how many times for each. Let's choose 3

3 fits

6 times → 6; 9 times → 9

$\frac{6}{9}$

Check whether you can simplify again. Yes 3 is also fits/is a common factor

$\frac{2}{3}$

Pick the highest common factor

$\frac{18}{27}$

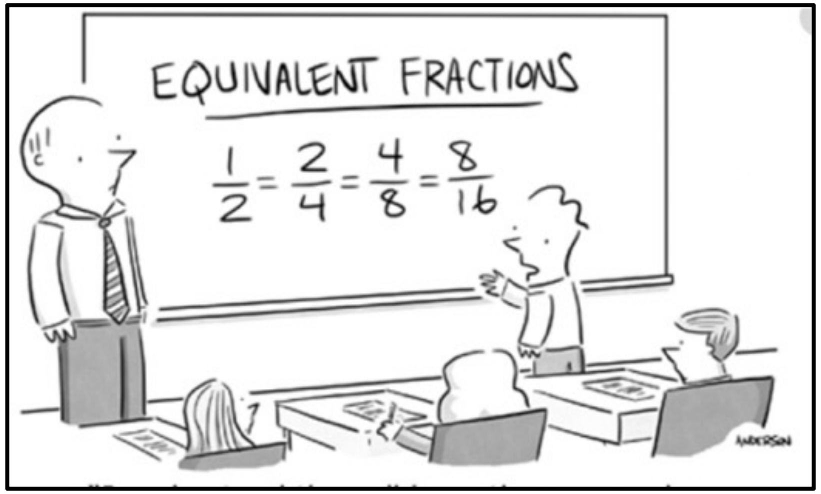
Find the highest common factor (not just any number that fits). 9 is the highest common factor

9 is the biggest number that fits

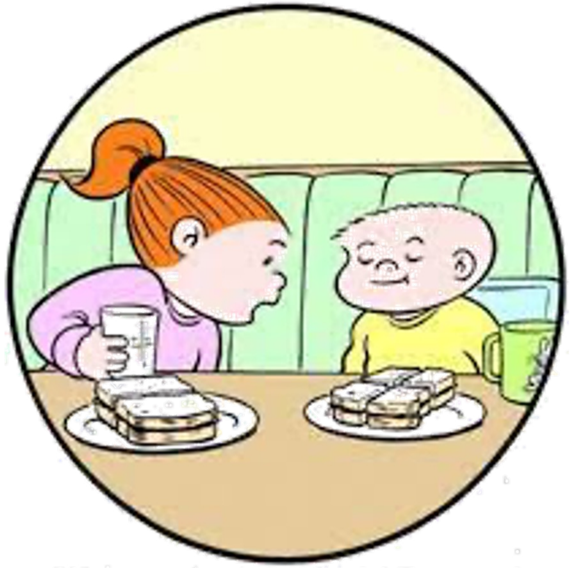
2 times → 2; 3 times → 3

$\frac{2}{3}$

Check whether you can simplify again. No, we are done since we chose the highest common factor which was 9



Equivalent Fractions



“Wait a minute! Why’d PJ get 4 sandwiches and I only got 2?”

Equivalent fractions are fractions that look different but have the same value when reduced/simplified into their lowest terms. You may **only multiply or divide** to get an equivalent fraction. Remember, whatever you do to the numerator you must do to the denominator.

Example 1 $\frac{18}{24} = \frac{3}{?}$

We concentrate on both the numerators first since we have both of them.

The numbers get smaller hence we divide

Step ① : Ask ourselves what do we divide 18 by to get 3? We divide by 6

$\div 6$

$$\frac{18}{24} = \frac{3}{4}$$

$\div 6$

Step ② : We must do the same here. So we divide 24 by 6 which is 4

$$\frac{3}{4}$$

Example 2 $\frac{2}{5} = \frac{?}{15}$

We concentrate on both the denominators first since we have both of them.

The numbers get bigger hence we multiply

Step ② : We must do the same here. So, we multiply 2 by 3 which is 6

$\times 3$

$$\frac{2}{5} = \frac{6}{15}$$

$\times 3$

Step ① : Ask ourselves what do we multiply 5 by to get 15? We multiply by 3

$$\frac{6}{15}$$

Example 3 $\frac{15}{20} = \frac{?}{4}$

We concentrate on both the denominators first since we have both of them.

The numbers get smaller hence we divide

Step ② : We must do the same here. So, we divide 15 by 5 which is 3

$\div 5$

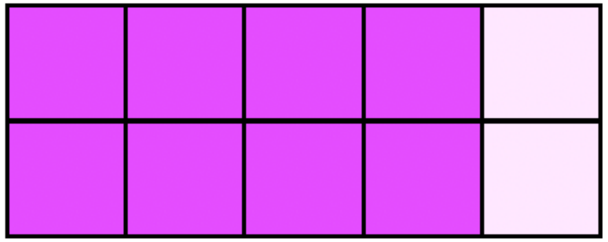
$$\frac{15}{20} = \frac{3}{4}$$

$\div 5$

Step ① : Ask ourselves what do we divide 20 by to get 4? We divide by 5

$$\frac{3}{4}$$

$\frac{8}{10}$ of the grid is shaded. Circle three fractions that are the same as $\frac{8}{10}$



$\frac{3}{4}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{14}{20}$ $\frac{16}{20}$ $\frac{80}{100}$

Answer

$$\frac{8}{10} = \frac{4}{5}$$

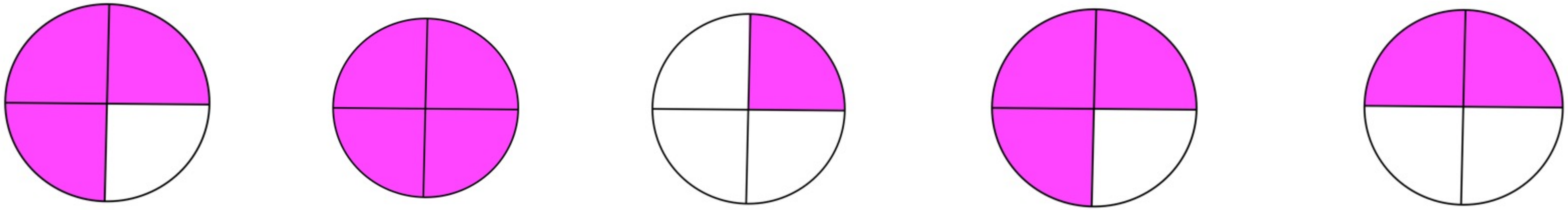
Simplifying all other fractions

$\frac{3}{4}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{7}{10}$ $\frac{4}{5}$ $\frac{4}{5}$

Going back to our originals

$\frac{3}{4}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{14}{20}$ $\frac{16}{20}$ $\frac{80}{100}$

Match the following fractions to their graphs $\frac{2}{8}$ $\frac{4}{8}$ $\frac{7}{7}$ $\frac{3}{4}$ $\frac{15}{20}$

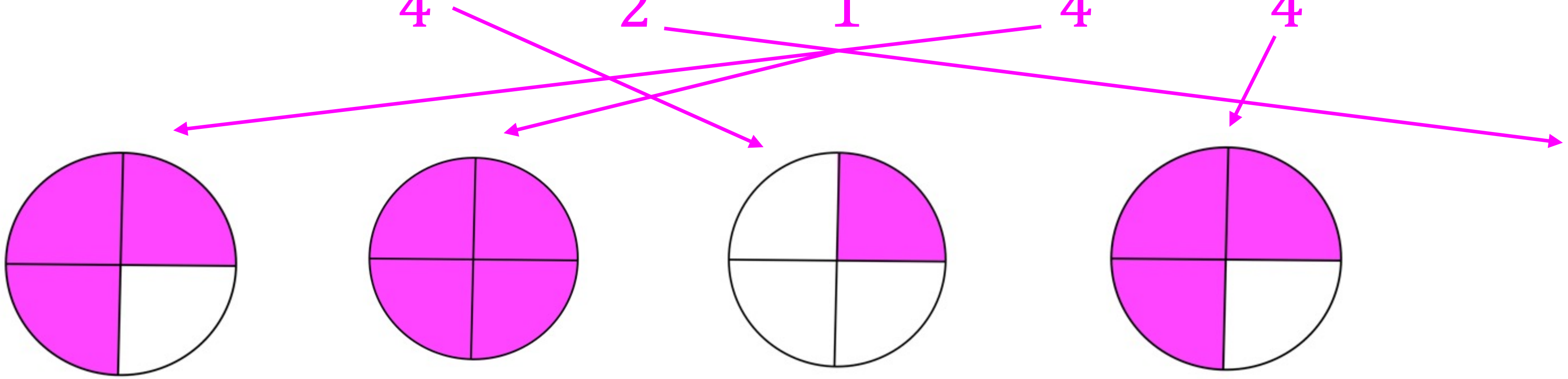


Answer

Simplify and then match

$\frac{2}{8}$ $\frac{4}{8}$ $\frac{7}{7}$ $\frac{3}{4}$ $\frac{15}{20}$

$\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{1}$ $\frac{3}{4}$ $\frac{3}{4}$



Fill in the missing parts $\frac{2}{3} = \frac{?}{9} = \frac{12}{?} = \frac{?}{21}$

Answer

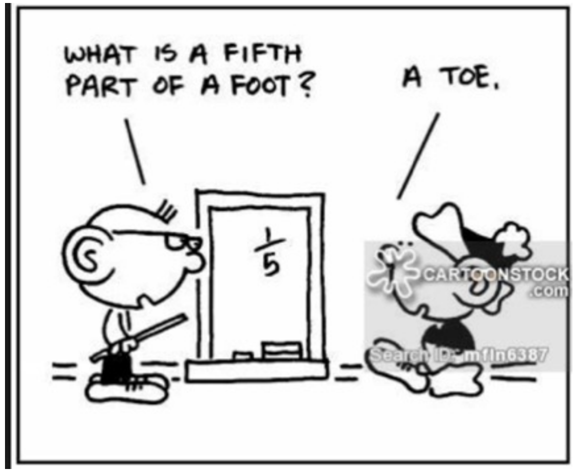
Pick one pair at a time to work on

blue pair $\frac{2}{3} = \frac{6}{9} = \frac{12}{?} = \frac{?}{21}$

purple pair $\frac{2}{3} = \frac{6}{9} = \frac{12}{18} = \frac{?}{21}$

pink pair $\frac{2}{3} = \frac{6}{9} = \frac{12}{18} = \frac{14}{21}$

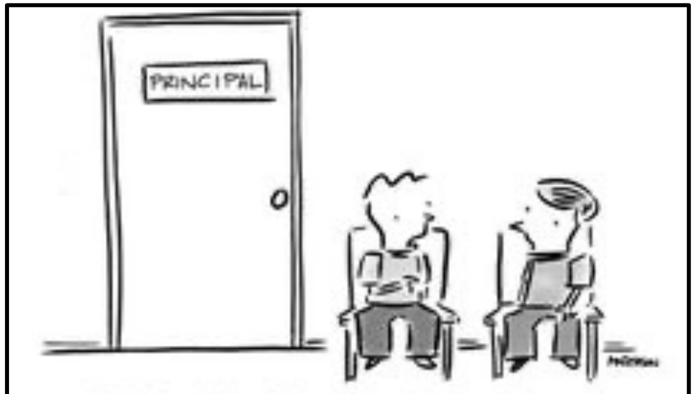
$$\frac{2}{3} = \frac{6}{9} = \frac{12}{18} = \frac{14}{21}$$



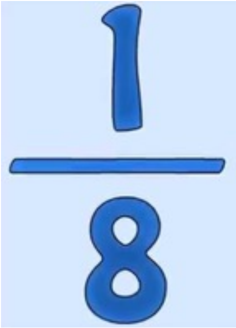
"I'd like to meet you halfway, but I'm terrible with fractions."

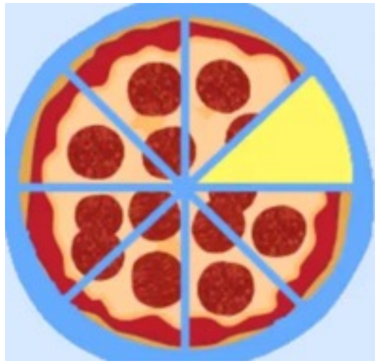
"To show you how well I understand fractions, I only did half of my homework."

Fractions Of An Amount



"I'd plead the fifth, but we haven't learned fractions yet."



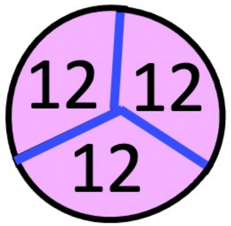


Let's first get some insight/intuition:

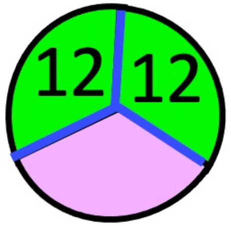
Penny has a drawer containing 36 socks. $\frac{2}{3}$ of them are green socks. How many green socks are in the drawer?

We slice 36 (the whole) into 3 pieces and then find what 2 of those pieces represent

cut into 3 pieces



We want 2 of them



$12 + 12 = 24$ green socks

Hindsight:

We just divide the total by the denominator first and then multiply by the numerator

Also realise that of just means multiply, so we are doing:

$$\frac{2}{3} \text{ of } 36$$

$$= \frac{2}{3} \times 36$$

$$= \frac{2}{3} \times \frac{36}{1}$$

$$= 24$$

So it should make sense that we can write our steps as:

find $\frac{\square}{\square}$ of $\square$

Step 1: Divide the total amount by the denominator

Step 2: Multiply the answer to step 1 by the numerator

Example 1

$find\ \frac{2}{5}\ of\ 15$

Step ① : Divide 15 by 5 which is 3

Step ② : Multiply 3 by 2

Step 1

$15 \div 5 = 3$

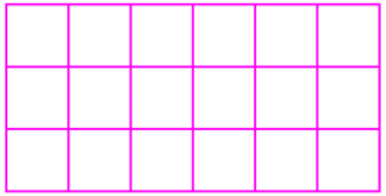
Step 2

3×2

$= 6$

Example 2

Shade $\frac{5}{6}$ of the following shape

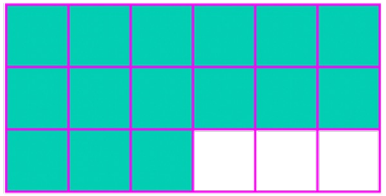


Long Method: Let's find what $\frac{1}{6}$ of the shape would be

We have 18 pieces so $\frac{1}{6}$ is the same as $\frac{18}{6} = 3$

We need to shade $\frac{5}{6}$ so we need 5 times what $\frac{1}{6}$ was

$5(3) = 15$ so we shade 15 pieces

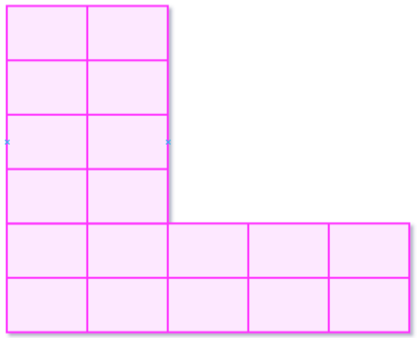


Short Method:

This question is just saying find $\frac{5}{6}$ of 18 and shade that number

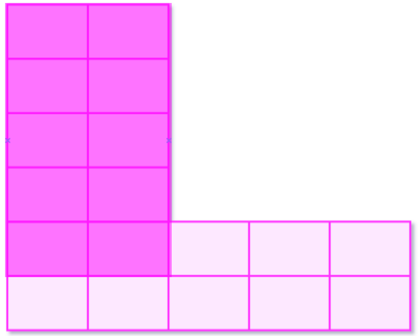
More examples:

Shade $\frac{5}{9}$ of the following shape

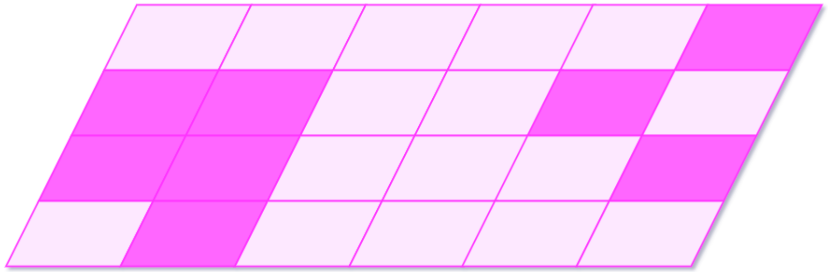


18 pieces

$\frac{5}{9}\,of\ 18 = 10$ pieces



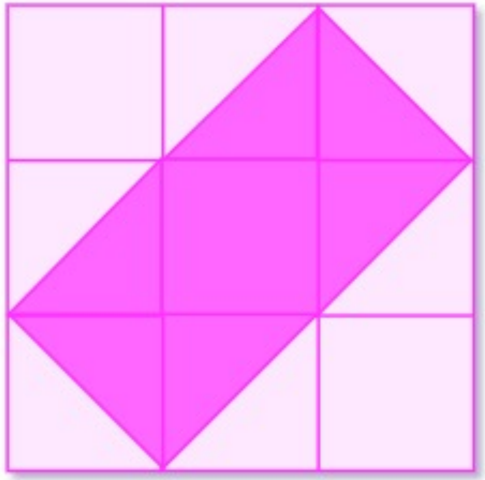
What fraction is shaded in each shape below



24 pieces

8 shaded pieces

$$\frac{8}{24} = \frac{1}{3}$$



2 triangles make a square

9 squares

4 shaded squares

$$\frac{4}{9}$$

Find $\frac{1}{2}\,of\ \frac{1}{3}\,of\ \frac{1}{4}\ of\ 48$

Work from right to left

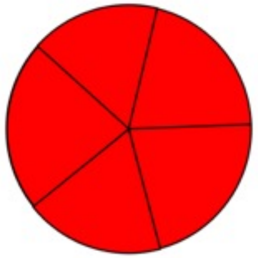
$$\frac{1}{2}\,of\ \frac{1}{3}\,of\ \frac{1}{4}\ of\ 48 = \frac{1}{2}\,of\ \frac{1}{3}\,of\ 12 = \frac{1}{2}\,of\ \frac{1}{3}\,of\ 12 = \frac{1}{2}\,of\,4 = 2$$

Harder Type: What do we do when given a fraction of an amount, but **not given** the **total**

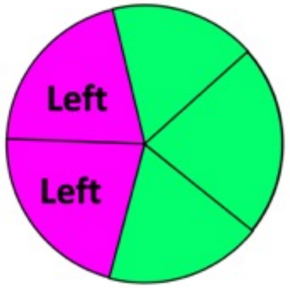
Belinda ate $\frac{3}{5}$ of a bar of chocolate. 60 grams of chocolate remained for Isabella. What was the original mass of the chocolate bar?

We do we not find $\frac{3}{5}$ of 60? Why? Because 60 is **not the total**!

Draw a pizza - we have 5 pieces in total since our fraction is out of 5



Belinda eats 3 pieces and 2 pieces are left so these 2 pieces must represent 60 grams



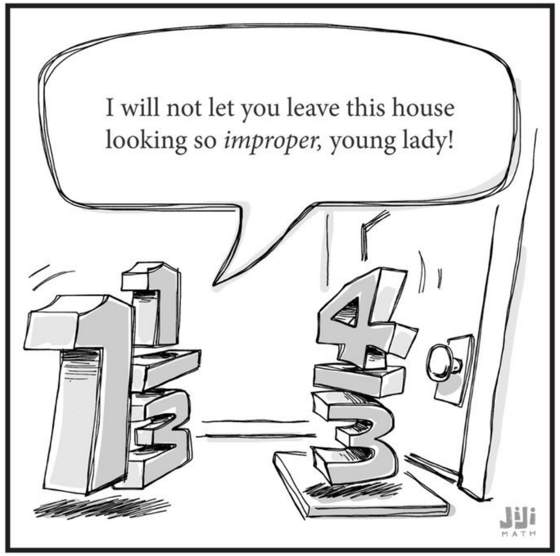
$$2 \text{ pieces} = 60$$

$$1 \text{ piece} = \frac{60}{2} = 30$$

We want the original mass which is all 5 pieces

$$5(30) = 150 \text{ grams}$$

Improper Fractions and Mixed Numbers



There are 3 Types Of Fractions

Proper Fractions

$$\frac{\text{smaller number}}{\text{bigger number}}$$

Improper Fraction

$$\frac{\text{bigger number}}{\text{Smaller number}}$$

Mixed Fraction

$$\text{whole number}\ \frac{\square}{\square}$$

- **Improper fractions** are fraction which have a numerator which is equal to or greater than the denominator. This means that the fraction would be greater than one if it was converted into a decimal. They can also be known as top-heavy fractions.

Improper Fraction looks like $\dfrac{\text{bigger number}}{\text{Smaller number}}$

- **Mixed fractions** or **mixed numbers** are a whole number and proper fraction which go together. They are always greater than one when they are converted into decimals.

Mixed Number looks like $\text{whole number}\dfrac{\square}{\square}$

Improper fractions can be converted into mixed fractions and vice versa.

Converting an improper fraction to a mixed number

$$\frac{\text{bigger number}}{\text{Smaller number}} = \text{How many times smaller number fits exactly into bigger number}\ \frac{\text{remainder}}{\text{original denominator}}$$

:

Step 1: Divide the numerator by the denominator

Step 2: Write down the whole number part of the answer

Step 3: Put the remainder in the numerator. The new denominator is the same as that of the original improper fraction

Example: Turn $\frac{17}{5}$ into a mixed number

5 fits into 17 3 times exactly

There is a remainder of 2 from 15 to 17

We copy the original denominator

$3\frac{2}{5}$

Converting a mixed number to an improper fraction

$$1\frac{2}{3} = \frac{1 \times 3 + 2}{3}$$

Step 1: Multiply the whole number by the denominator

Step 2: Add the answer to step 1 to the numerator

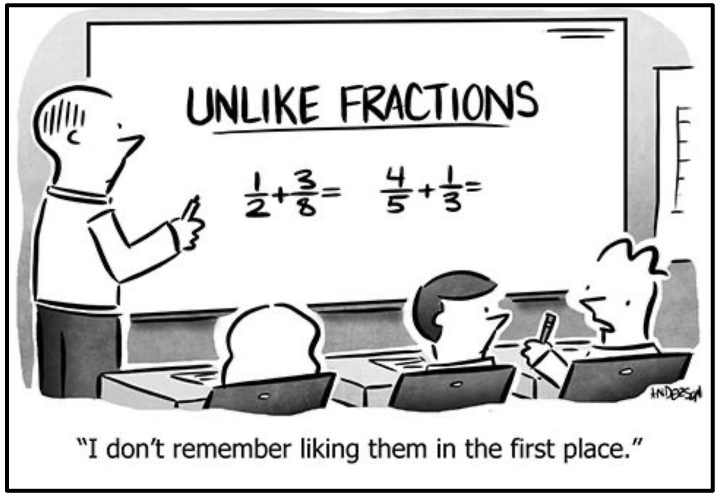
Step 3: Write the result of step 2 in the numerator

Step 4: Copy the original denominator

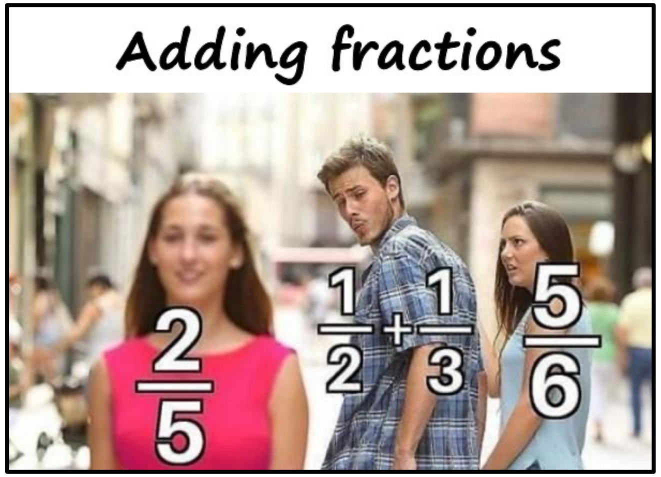
Example: Turn $7\frac{3}{8}$ into an improper fraction

$$= \frac{7 \times 8 + 3}{8}$$

$$= \frac{59}{8}$$



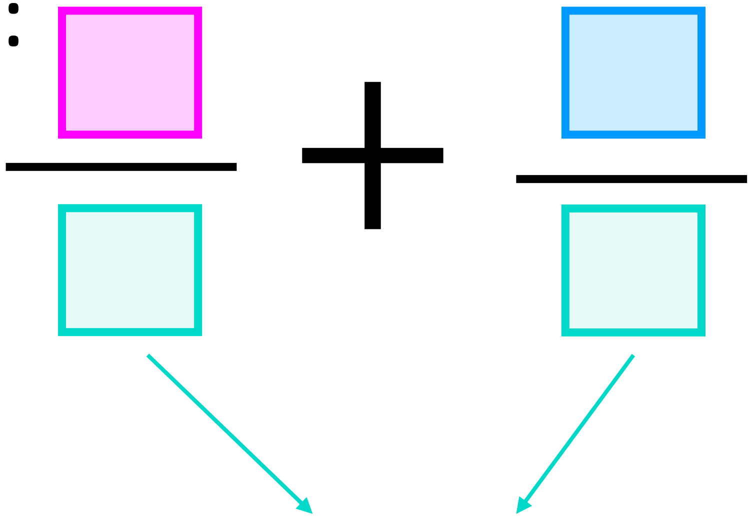
"I don't remember liking them in the first place."



Adding and Subtracting Fractions

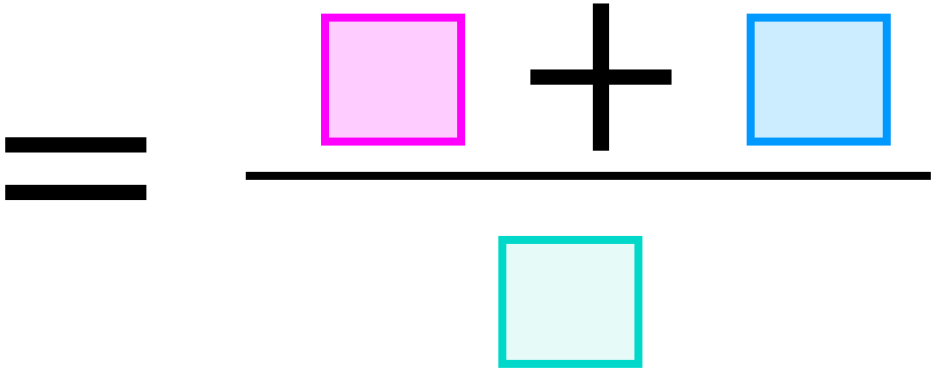
When we add or subtract fractions, we want the denominators to be the same

Adding:

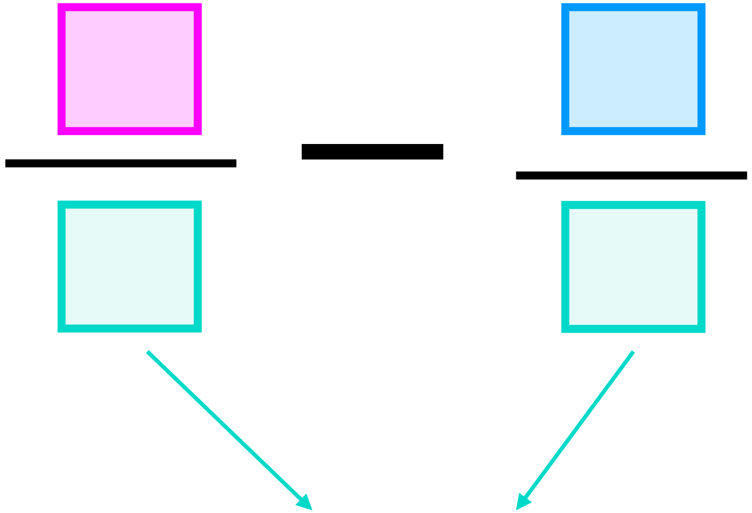


We want these to be the same which they are (indicated by the same colour)

Then we simply add the top numbers and keep the bottom numbers the same

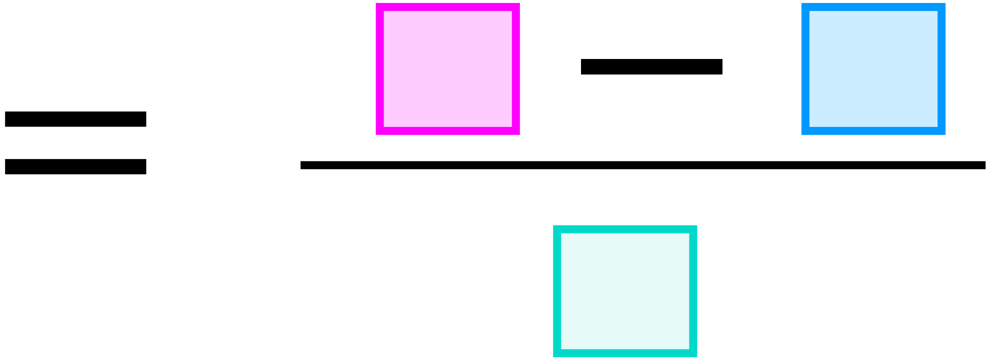


Subtracting:



We want these to be the same which they are (indicated by the same colour)

Then we simply add the top numbers and keep the bottom numbers the same

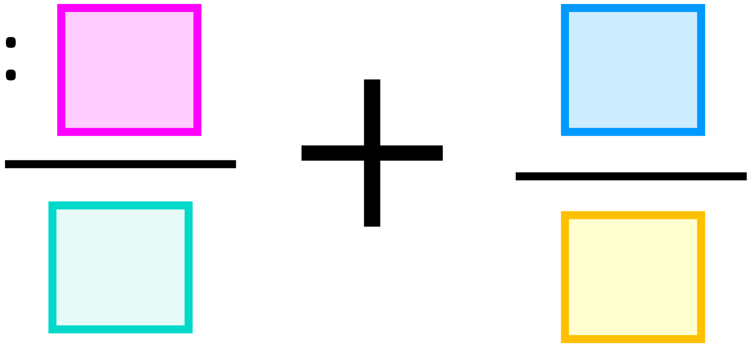


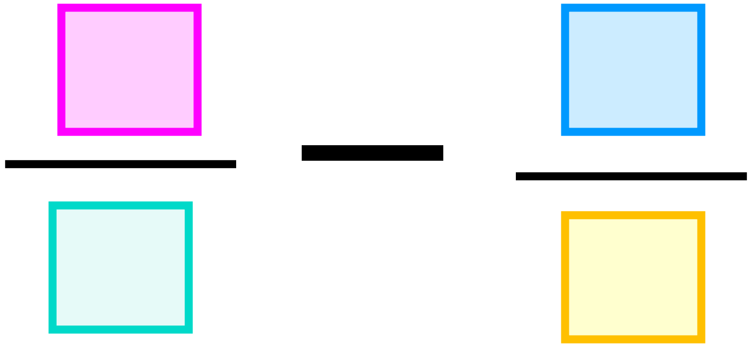
If the denominators are already the same, then we just add/subtract the numerators and **keep the denominators the same**.

For example,

$$\frac{2}{7}+\frac{3}{7}=\frac{2+3}{7}=\frac{5}{7}$$

If the denominators are not the same such as (different colours indicate different numbers)

Adding: 

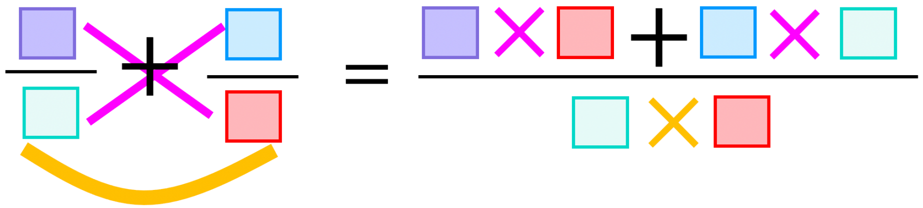
Subtracting: 

We have to make them the same, which we will see how to do over the next few pages

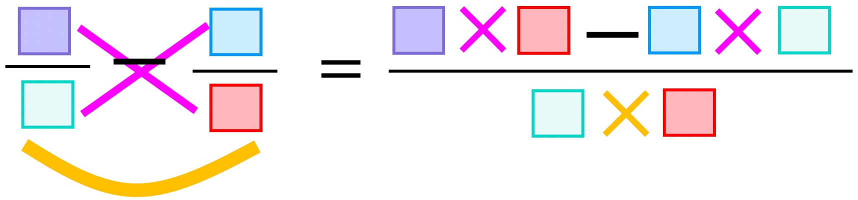
If the denominators are NOT the same we have to make them the same which is known as a **common denominator.** Then there are 2 ways to deal with fractions:

Way 1: Kiss and Smile

Adding



Subtracting



Consider $\frac{2}{3}+\frac{1}{6}$

$$\frac{2}{3}+\frac{1}{6}=\frac{2\times 6+1\times 3}{3\times 6}=\frac{12+3}{18}=\frac{15}{18}=\frac{5}{6}$$

Way 2: Find A Common Denominator

The common denominator is the lowest common multiple, which is the smallest number that that both the denominators fit into. For example:

The multiples of 4 are 4,8,12,16,20,24,28,32,36,40,44....

The multiples of 5 are 5,10,15,20,25,30,35,40,50,60,70......

The **common multiples** of 4 and 5 are those that are found in both lists.

4,8,12,16,20,24,28,32, 36,40,44,...

5,10,15,20,25,30,35,40,50,50,...

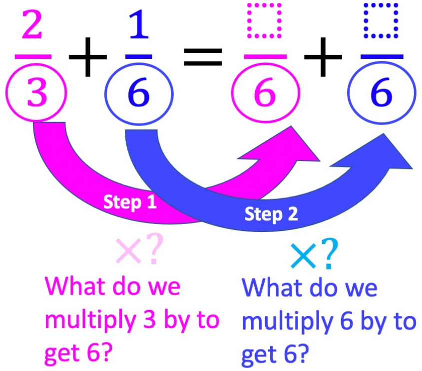
The **lowest** common multiple is the smallest of the common multiples which is 20.

Consider $\frac{2}{3}+\frac{1}{6}$

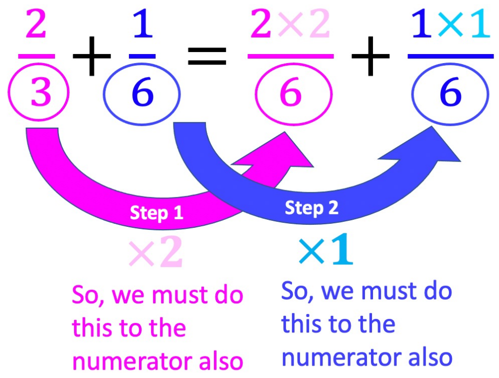
The lowest common multiple of 3 and 6 is 6 so we write both fractions over 6.

$$\frac{2}{3}+\frac{1}{6}=\frac{\square}{6}+\frac{\square}{6}$$

We need to find the new numerators. To do this we need to write EACH colour pair below as an **equivalent fraction**



$\Longrightarrow$



We end up with $\frac{2\times 2}{6}+\frac{1\times 1}{6}=\frac{4}{6}+\frac{1}{6}=\frac{5}{6}$

A Few Things To Be Aware Of

Note: The method on the right on the previous page is better as using the method on the left since the method on the left sometimes produces bigger denominators (such as the example) and means we have to simplify further.

Even once students know the technique for adding and subtracting they sometimes get stuck when

- One of the numbers number is a whole number and the other is a fraction
- Mixed numbers appear

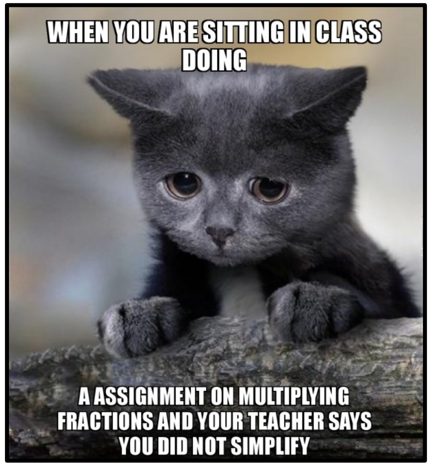
Tips for each case:

- If we have a whole number then write it over 1, so that both are fractions. Why do we do this? It is easier to deal with two fractions rather than a whole number and a fraction

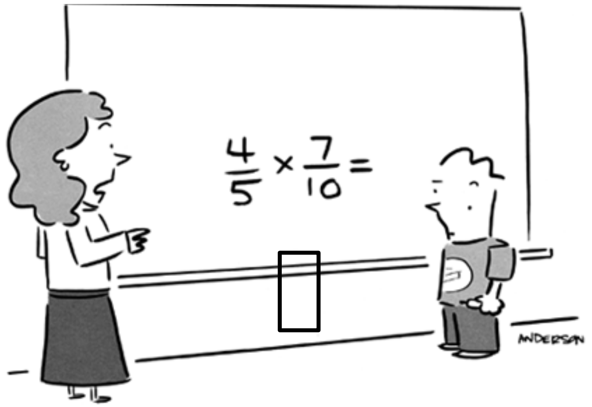
  For example, $2\times\frac{3}{4}=\frac{2}{1}\times\frac{3}{4}=\frac{6}{4}=\frac{3}{2}$

- If you have a mixed number, turn it into an improper fraction and proceed as normal

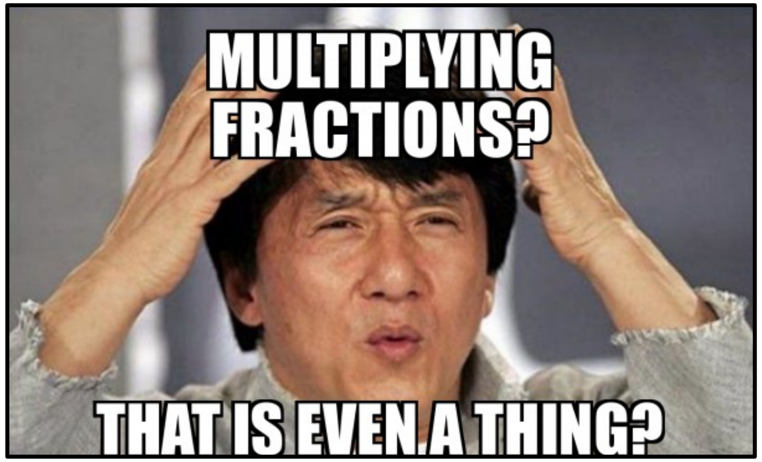
  For example, $2\frac{1}{8}\times\frac{3}{4}=\frac{17}{8}\times\frac{3}{4}=\frac{51}{32}$



Multiplying Fractions

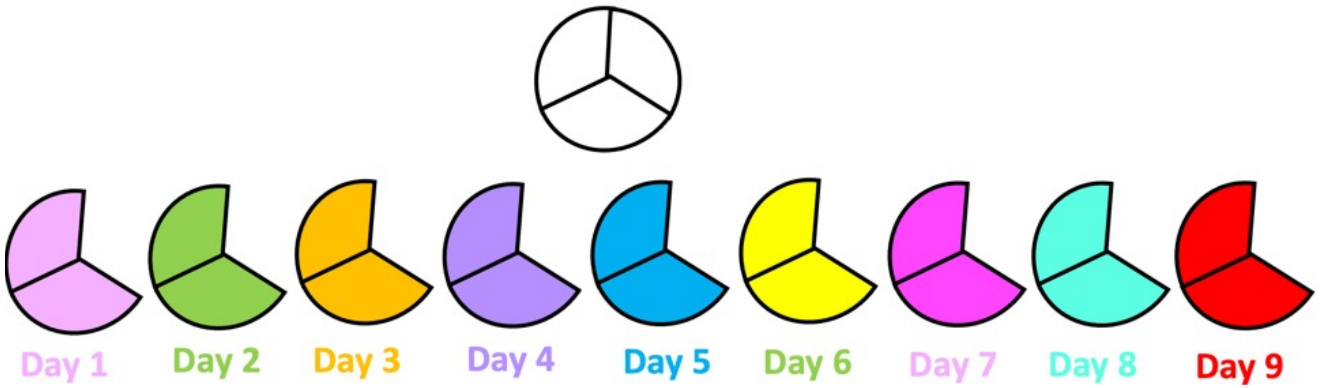


"Actually, ninjas multiply fractions all the time. They just never talk about it. Because they're ninjas."

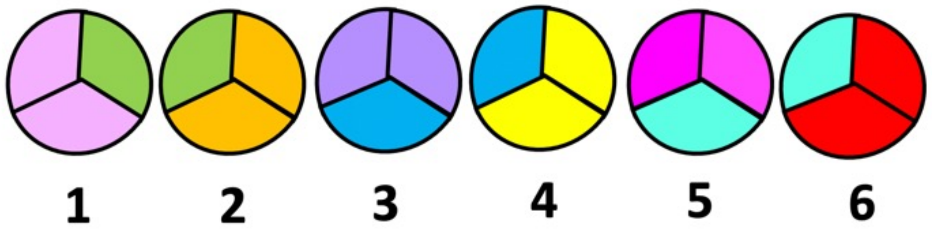


Let’s see how multiplication works with an example and a picture.

I eat $\frac{2}{3}$ of a pizza each day. How many pizzas do I eat after 9 days?



Let’s put these together and see how many circles these make



6 perfect circles and hence 6 pizzas

There is a shortcut though. We don’t have to draw these out each time.

We just multiply across when we multiply fractions

In other words

Multiply the numerators together to get the new numerator

Multiply the denominators together to get the new denominator

As mentioned we **don't need a common denominator** when we multiply fractions, unlike with adding and subtracting above. We can find one, but it just makes our numbers unnecessarily big. We simply multiply across when we multiply fractions (multiply the numerator with the numerator and the denominator with the denominator) like the colours show:

We might be able to cancel though before multiplying if the numbers are big. Let's look at how in detail.

Step 1: Try and cancel. We can cancel by simplifying diagonally or vertically, but NOT horizontally.
We don't have to cancel first, but it makes the numbers smaller and hence more manageable.

can cancel diagonally

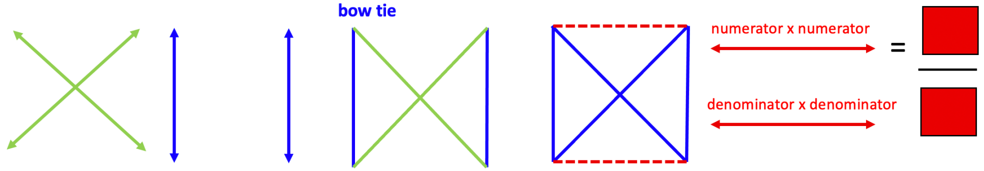
can also cancel vertically

you can remember the cancelling directions (vertical and horizontal) by the fact that they form a bow tie

DO NOT cancel horizontally

We multiply across horizontally to get answer

For example, we can either cancel the colour sets, in each of the examples below:



Option 1: Cancel Vertically

$$\frac{16}{12} \times \frac{18}{20} = \frac{4}{3} \times \frac{9}{10}$$

We cancelled (simplified) the green pair vertically $\frac{16}{12} = \frac{4}{3}$

and also the purple pair vertically $\frac{18}{20} = \frac{9}{10}$

We could cancel again but the numbers are more manageable, so we don't really need to. Let's see how we would do this though.

$\frac{4}{3} \times \frac{9}{10}$ can be cancelled diagonally now to give $\frac{2}{1} \times \frac{3}{5}$

Option 2: Cancel Diagonally

$$\frac{16}{12} \times \frac{18}{20} = \frac{4}{2} \times \frac{3}{5}$$

We cancelled (simplified) the blue pair diagonally $\frac{18}{12} = \frac{3}{2}$ and also

the pink pair diagonally $\frac{16}{20} = \frac{4}{5}$

We could cancel again but the numbers are more manageable, so we don't really need to. Let's see how we would do this though.

$\frac{4}{2} \times \frac{3}{5}$ can be cancelled horizontally on the first fraction to give $\frac{2}{1} \times \frac{3}{5}$

Step 2: Multiply across horizontally for the numerator and then the same for the denominator. For example, $\frac{2}{1} \times \frac{3}{5} = \frac{2 \times 3}{1 \times 5} = \frac{6}{5}$

Let’s look at another example:

$$\frac{5}{3}\times\frac{8}{15}$$

Way 1: Without cancelling first

$$\frac{5}{3}\times\frac{8}{15}=\frac{5\times 8}{3\times 15}=\frac{40}{45}=\frac{8}{9}$$

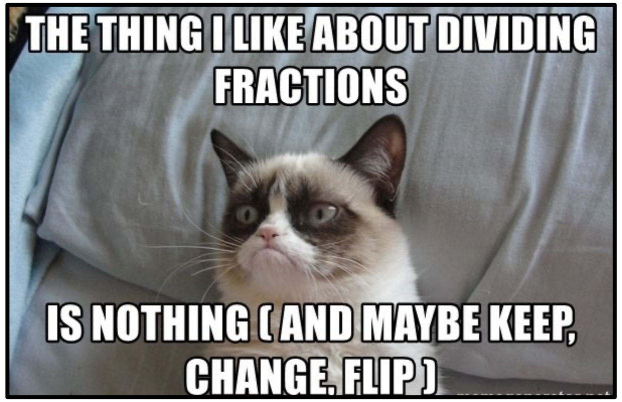
Way 2: With cancelling first (the 5 and 15 diagonally)

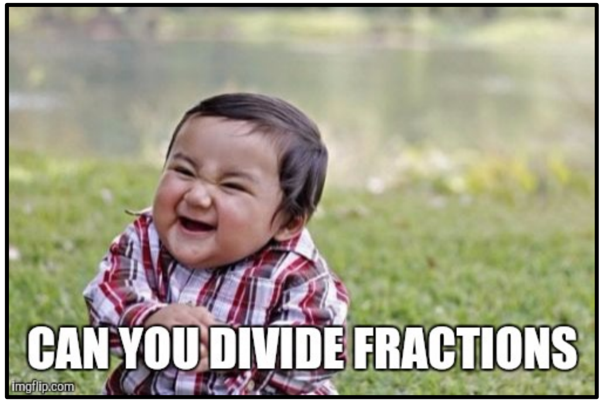
$$\frac{5}{3}\times\frac{8}{15}=\frac{1}{3}\times\frac{8}{3}=\frac{8}{9}$$

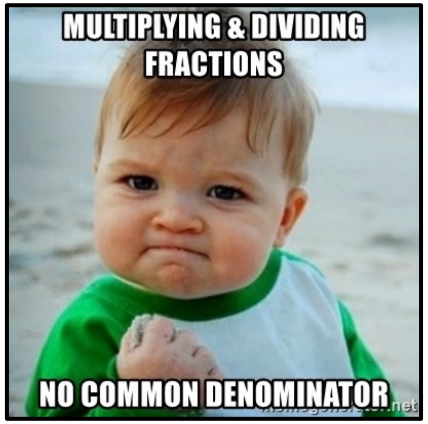
Way 3: With getting a common denominator (not necessary) and then multiply across

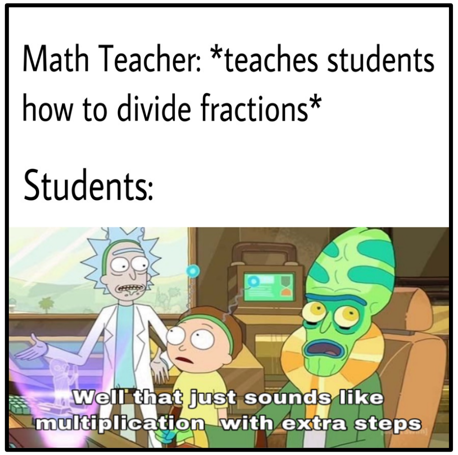
$$\frac{5}{3}\times\frac{8}{15}=\frac{25}{15}\times\frac{8}{15}=\frac{25\times 8}{15\times 15}=\frac{200}{225}=\frac{8}{9}$$

Notice how this way took longer. I have only shown this because students are sometimes tired of so many rules to remember. If studenst are used to making common denominators, doing it for multiplication won't hurt.

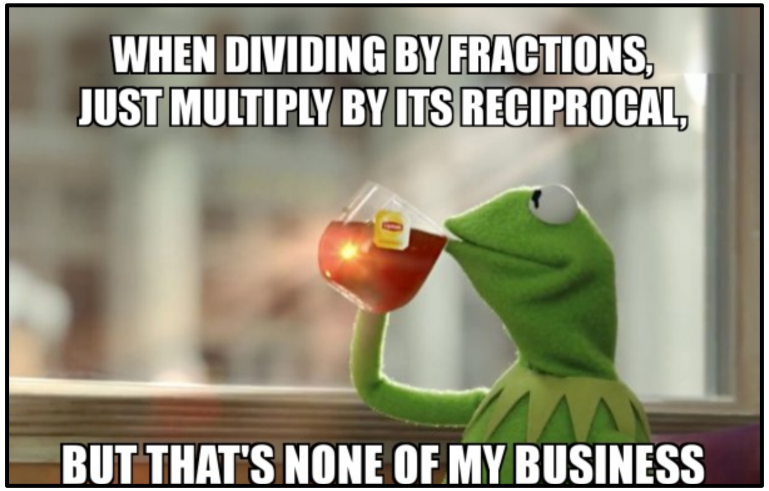


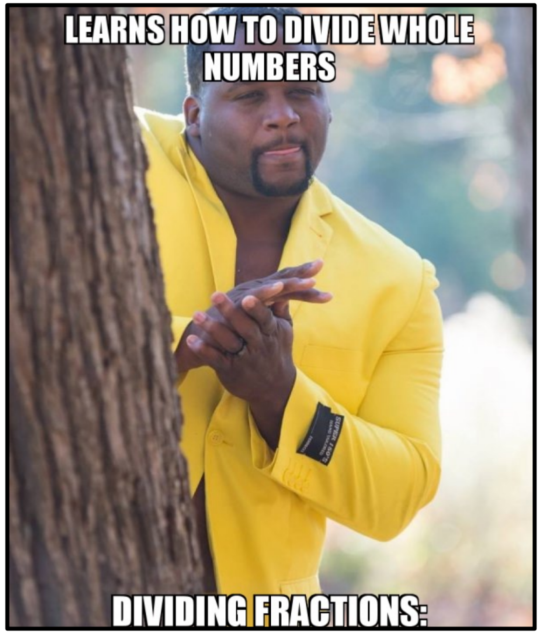


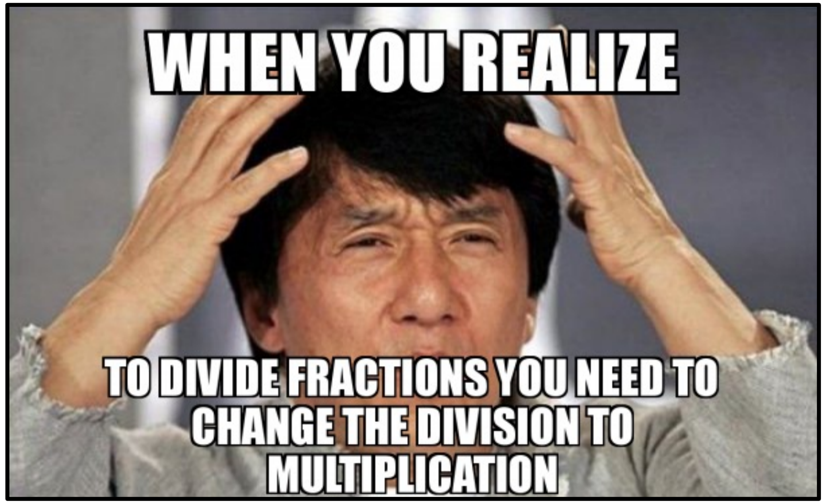




Dividing Fractions





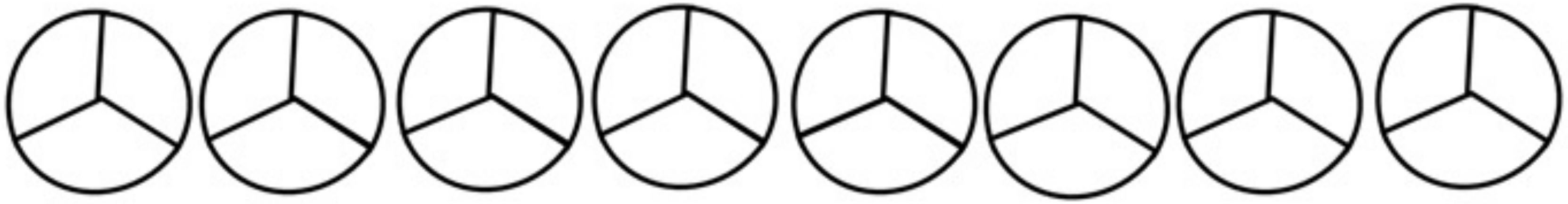


There is a keep, change, flip step by step method for dividing fractions which we will cover in a bit. But first of all, lets get some intuition to prove that “keep change flip” is not just some “magic”.

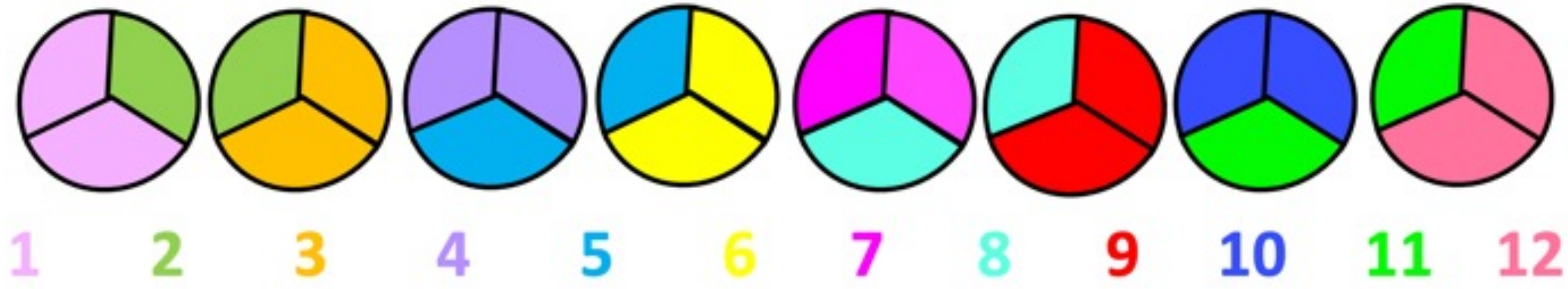
Jeff has 8 pizzas to share between himself and friends. If each person is to get $\frac{2}{3}$ of a pizza, how many friends can he feed?

Firstly, we draw 8 pizzas

Let’s cut each into slices of 3 since the fraction $\frac{2}{3}$ is out of 3



Let’s see how many $\frac{2}{3}$ we can fit (each colour represents $\frac{2}{3}$)



We can feed 12 friends

Let’s look at another example

Dividing fractions is the same as whole number division. If we have the very basic division problem of 8 divided by 2, we are trying to determine how many groups of 2 are in the number 8.

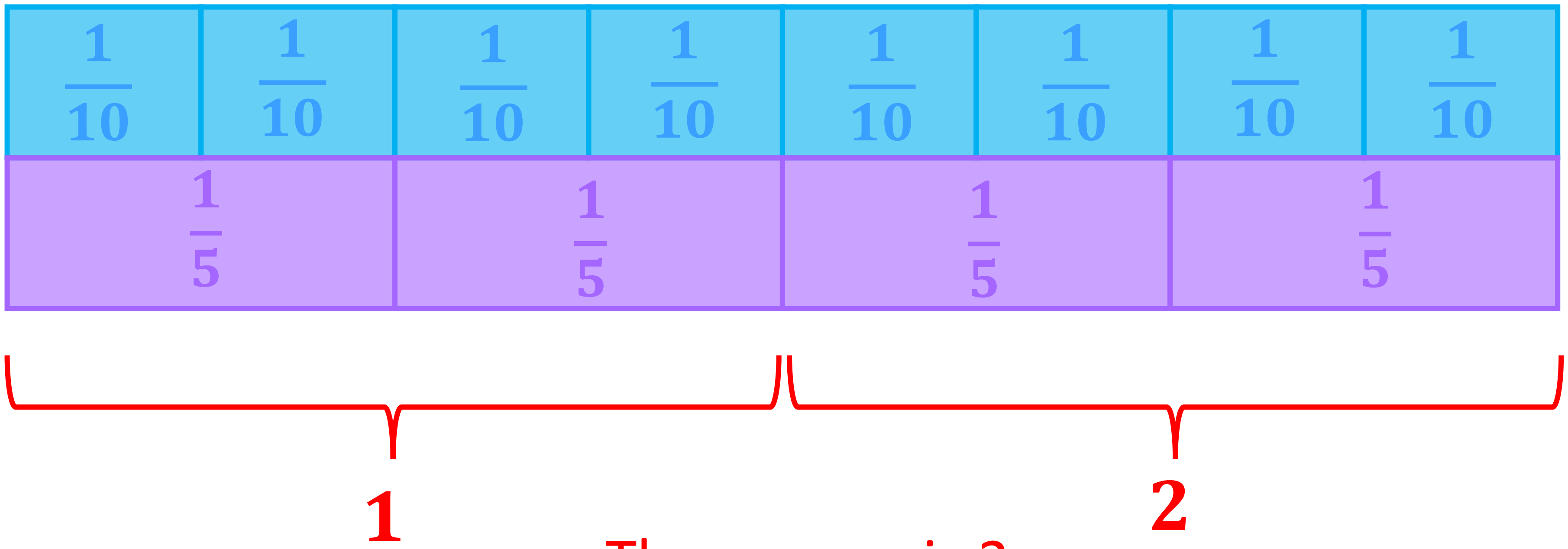
It’s the same with fractions. Consider

$$\frac{8}{10} \div \frac{2}{5}$$

“how many groups of $\frac{2}{5}$ can be made out of $\frac{8}{10}$”?

This just says “how many groups of $\frac{2}{5}$ cover $\frac{8}{10}$”?

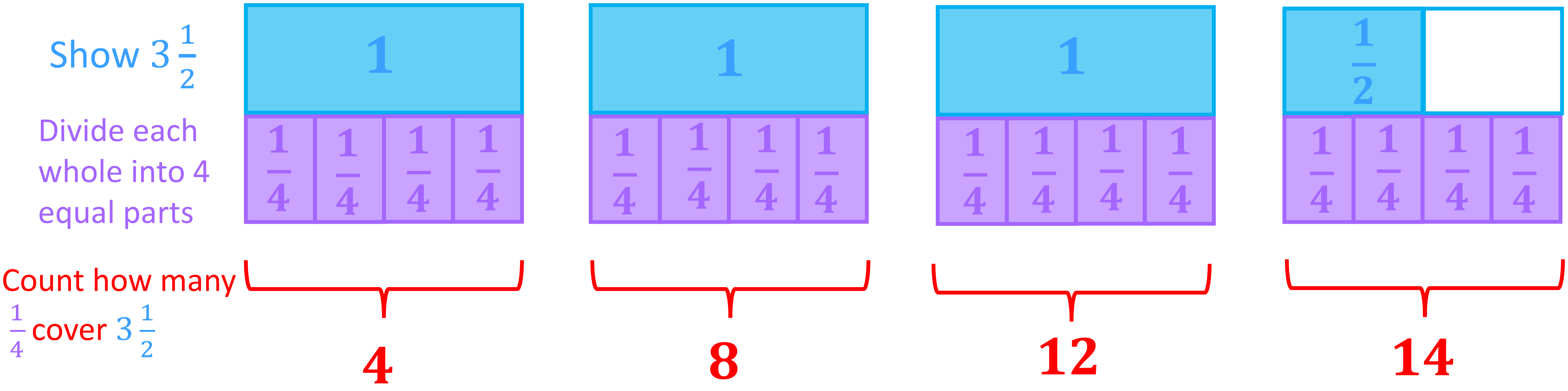
In other words, how many $\frac{2}{5}$ are in $\frac{8}{10}$?



The answer is 2

$$3\frac{1}{2} \div \frac{1}{4}$$

How many groups of $\frac{1}{4}$ are there in $3\frac{1}{2}$?



The answer is 14

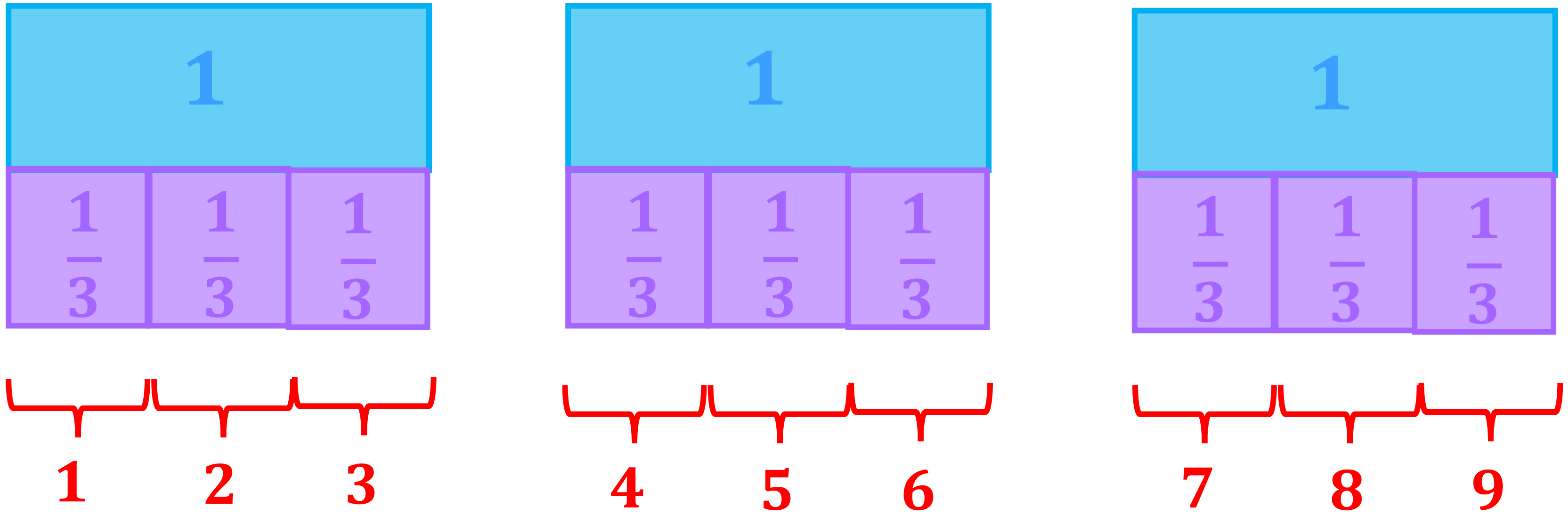
$3 \div \frac{1}{3}$

How many groups of $\frac{1}{3}$ make 3 wholes?

Draw 3 wholes

Divide each into 3 equal parts

Count how many $\frac{1}{3}$ cover 3 wholes



The answer is 9

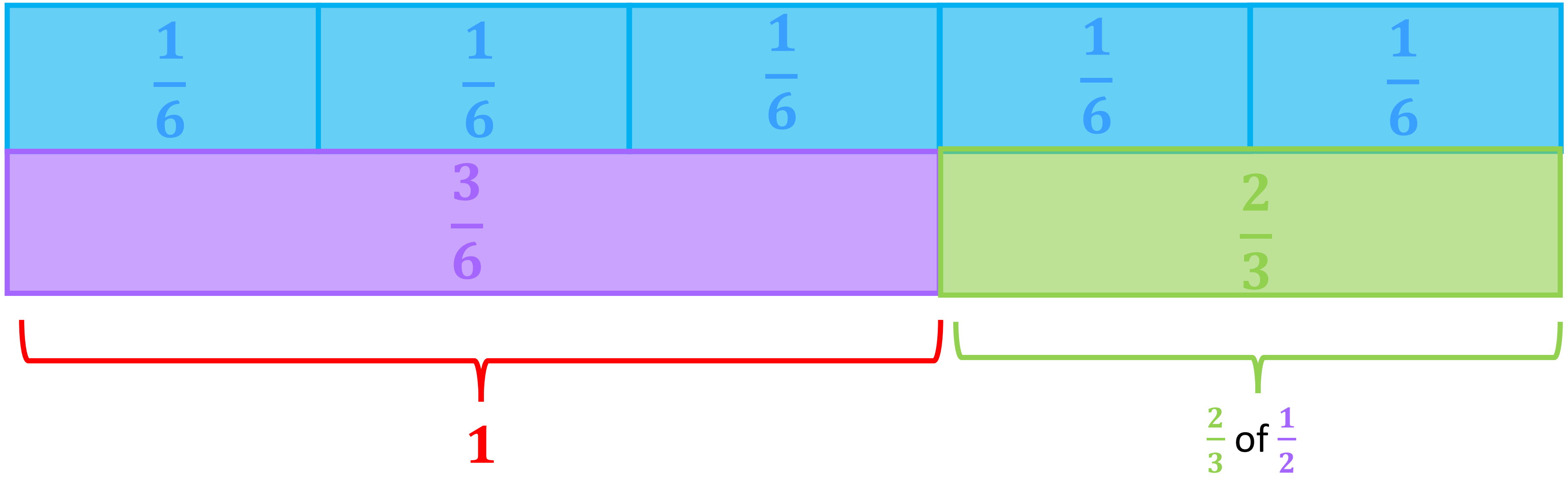
Let's look at a harder example

$$\frac{5}{6} \div \frac{1}{2}$$

This is asking how many $\frac{1}{2}$'s fit into $\frac{5}{6}$. It's easier to see how many half's fit if we use an equivalent fraction for $\frac{1}{2}$ to make sixths. $\frac{1}{2} = \frac{3}{6}$. So instead of how many halves fit now, it's how many groups of $\frac{3}{6}$ fit in $\frac{5}{6}$?

We can only make 1 group of $\frac{3}{6}$ out of $\frac{5}{6}$, but there are 2 pieces out of the 3.

We needed to make a whole group so only $\frac{2}{3}$ of the $\frac{3}{6}$ are left so the answer is $1\frac{2}{3}$

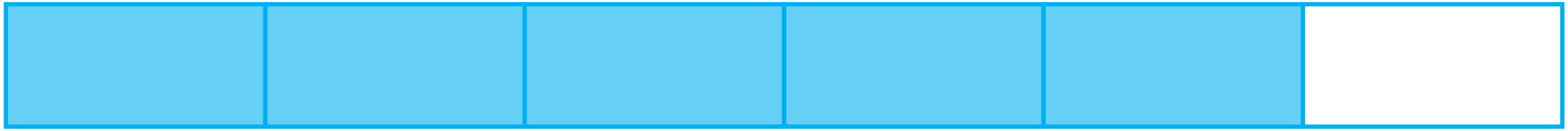


If we use multiplying by the reciprocal method: $\frac{5}{6} \div \frac{1}{2}$ is $\frac{5}{6} \times \frac{2}{1} = \frac{10}{6}$ or $\frac{5}{3}$ also known as 5 divided by 3 or $1\frac{2}{3}$.

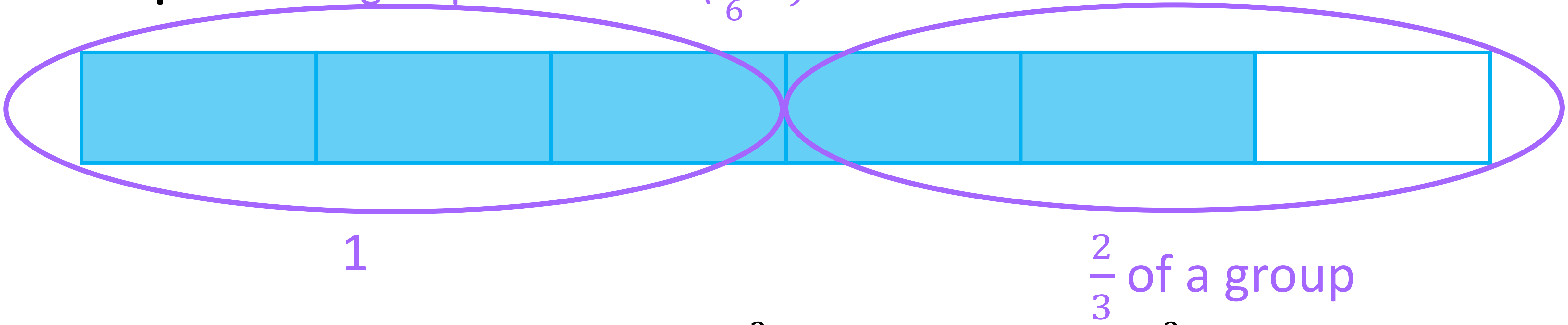
We could also have represented $\frac{5}{6} \div \frac{1}{2}$ in the following way:

Step 1: Convert $\frac{1}{2}$ to $\frac{3}{6}$ (Find common denominator)

Step 2: Draw dividend $\frac{5}{6}$



Step 3: Circle groups of divisor ($\frac{3}{6}'s$)



1 full/whole group fits plus $\frac{2}{3}$ of another group $\Longrightarrow 1\frac{2}{3}$

We don't need to draw models every time. There is a nice step-by-step short-cut method for dividing fractions which is to change the division sign into multiplication, flip the second fraction and then proceed above as for multiplication. This process is known as "keep change flip". Just like with multiplying fractions we also don't need a common denominator when dividing. So, we have

Step 1: Keep the first fraction

Step 2: Change the division sign to a multiplication sign

Step 3: Flip the second fraction

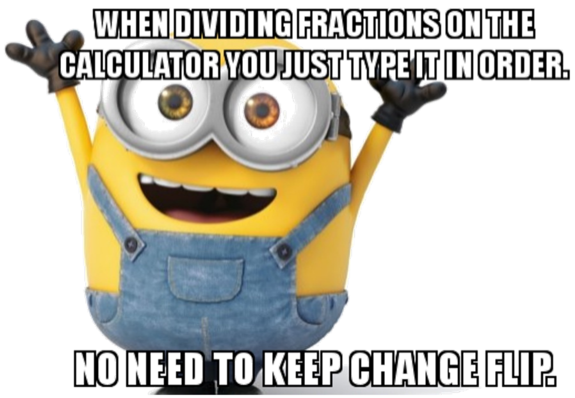
Step 4: Proceed as for multiplying fractions

Let's look at an example:

$$\frac{3}{4} \div \frac{9}{8}$$

Let's keep change and flip

$$\frac{3}{4} \times \frac{8}{9}$$

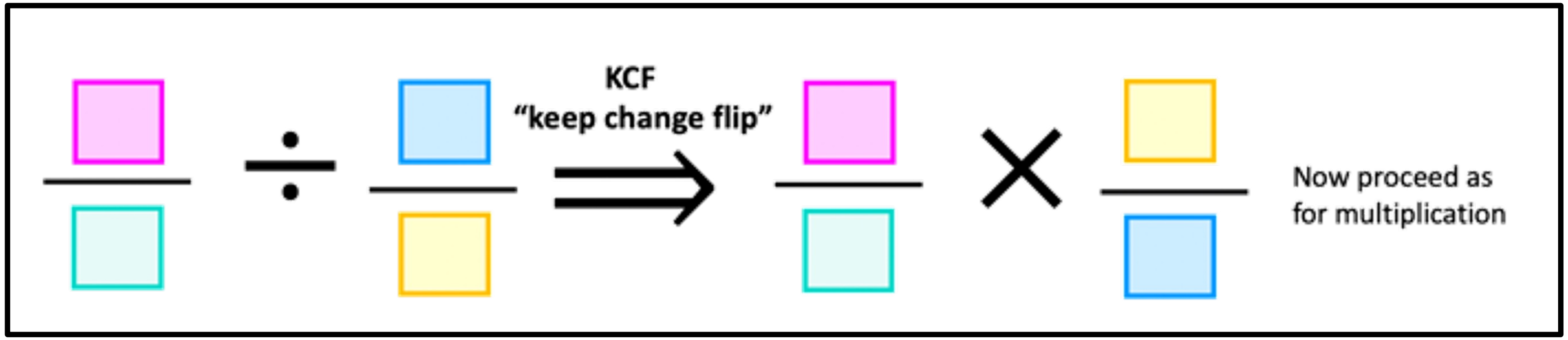


Now apply your knowledge of multiplying fractions. We can cross cancel first, but the numbers are small and manageable, so it is not necessary.

$$= \frac{24}{36} = \frac{2}{3}$$

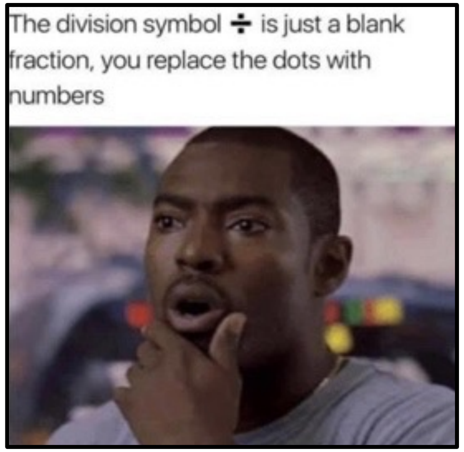
Example continued

$$\frac{3}{4} \div \frac{9}{8}$$



We saw that

$$\frac{3}{4} \div \frac{9}{8} = \frac{3}{4} \times \frac{8}{9} = \frac{24}{36} = \frac{2}{3}$$



A nice shortcut:

$$\frac{\frac{3}{4}}{\frac{9}{8}}$$

bottom times top

top times bottom $= \frac{3 \times 8}{4 \times 9} = \frac{24}{36} = \frac{2}{3}$

Although not necessary, instead of keep change flip and multiplying you can find a common denominator

$$\frac{5}{6} \div \frac{1}{2}$$

$$= \frac{5}{6} \div \frac{3}{6}$$

Now divide the numerators and the denominators

$$= \frac{5 \div 3}{6 \div 6} = \frac{5 \div 3}{1} = \frac{5}{3} = 1\frac{2}{3}$$

I have only shown this because students are sometimes tired of so many rules to remember. If they are used to making common denominators, doing it for division won't hurt.

So “keep change flip” is really not some magic

Division can be explained best in 3 different ways:

- Consider $10 \div 2$ and $10 \div \frac{1}{2}$. What even is division? Making groups of what we are dividing by! Consider 10 identical items and do $10 \div 2$. So, we can make 5 groups of 2. What is $10 \div \frac{1}{2}$? 10 groups of $\frac{1}{2}$. We are making groups of $\frac{1}{2}$. So, if we cut the objects in half we get 20 halves, meaning $10 \div \frac{1}{2}$. = 20. 20 halves not 20 wholes so $\frac{1}{2}$ is the same as multiplying by 2. $10 \div 2$ is the same as $10 \times \frac{1}{2}$.

- Consider the example $\frac{1}{3}$ divided by $\frac{2}{5}$. Rewrite with a common denominator so it is now $\frac{5}{15}$ divided by $\frac{6}{15}$. Then divide the numerators (like with multiplying we multiply the numerators and denominators separately). So $5 \div 6$ becomes $\frac{5}{6}$. This is the new numerator. Then divide the denominators: $15 \div 15$ becomes 1. Thus, the resulting complex fraction is $\frac{5}{6}$ over one, which is of course $\frac{5}{6}$. After doing several easy problems you should see the pattern of “keep change flip”.

- Dividing by 1 by 4 gives you $\frac{1}{4}$. Dividing a whole pizza by 1/8ths gives you 8. Notice that the division by a fraction $\frac{a}{b}$ is the same as multiplying by $\frac{b}{a}$.

Extra activities:

Get a measuring cups, fill a $\frac{1}{2}$ cup with something like (small dried beans, or something granular, etc). What happens if you divide the half cup by two? Pour the contents into 2 separate $\frac{1}{4}$ cups.

Research multiplicative inverse

- Dividing by 2 is the same as multiplying by ½ (the reciprocal)
- Dividing by 3 is multiplying by ⅓ (the reciprocal)

  etc

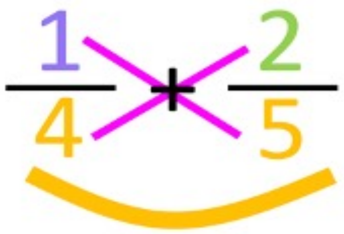
The opposite of division is multiplication and the opposite of multiplication is division

The flipping part of keep change flip is just multiplying by the reciprocal of the fraction!

Word Problem Example and Exercises

A group of students went to an outdoor centre. $\frac{1}{4}$ of them went canoeing and $\frac{2}{5}$ of them went sailing. The rest went mountain biking. What fraction of the group went mountain biking?

This question involves addition of fractions. We need to do $\frac{1}{4} + \frac{2}{5}$ first to find the total.

Way 1: Kiss and smile	Way 2: Find a common denominator
 $\frac{1 \times 5 + 2 \times 4}{4 \times 5} = \frac{5+8}{20} = \frac{13}{20}$	$\frac{1}{4} + \frac{2}{5}$ The common denominator is 20 $= \frac{?}{20} + \frac{?}{20}$ $= \frac{5}{20} + \frac{8}{20} = \frac{13}{20}$

Think of this as a pizza. We have eaten 13 slices out of 20 slices and the remaining 7 represent the rest

$\frac{7}{30}$ went mountain biking

Basics

A bag contains 5 cherry sweets and 10 lime sweets. What fraction of the sweets are cherry sweets?

Bethany cuts her birthday cake into 20 equal slices. She gives 16 slices to her friends. What fraction of the cake does Bethany have left?

A fruit punch is made up of 2 litres of orange juice, 3 litres of lemonade, 1 litre of cranberry juice and 4 litres of apple juice. What fraction of the fruit punch is orange juice?

$\frac{3}{5}$ of a class are girls. What percentage are boys?

Addition/Subtraction

Martha buys a ribbon that is $\frac{7}{10}$ of a yard long. She cuts $\frac{2}{5}$ off of a yard. How many yard of ribbon does Martha have left?

Chloe, Daisy, and Emily share some money. Chloe receives $\frac{5}{8}$ of the money, Daisey receives $\frac{1}{6}$ of the money. How much does Emily receive?

Multiplication

I eat $\frac{4}{5}$ of a pizza each day. How much pizzas do I eat after 55 days?

Division

At a party, 15 children share 3 identical cakes equally. What fraction of a cake does each child receive?

My cat eats two thirds of a bottle of milk a day. How long will 6 bottles take?

My cat Tabitha eats $\frac{3}{5}$ of a tin of cat food each day. How many days does a pack of 6 tins last her?

Julie shares 12 bars of chocolate amongst her pupils. Each pupil receives $\frac{2}{3}$ of a bar of chocolate and none is left over. How many pupils are there?

Fractions Of Amounts

Given Total:

Caitlin has 24 cupcakes. She eats $\frac{2}{3}$ of them. How many does she eat?

There are 35 people in total. $\frac{2}{5}$ are boys. How many are girls?

A book has 276 pages. Amina has read $\frac{1}{3}$ of the book. How many pages are left for Amina to read?

A box contains 48 pieces of fruit. Five eighths of them are apples and the rest of them are pears. How many pears are there?

A pizza is divided into 12 equal slices. Matthew eats $\frac{1}{4}$ of the pizza and Katie eats another $\frac{1}{6}$. How many slices are left?

Fractions Of Amounts Continued

There are 24 girls in a class. There are a quarter as many boys as girls in the class.
i. How many boys are in the class
ii. How many boys and girls are in the class?

Amber buys 4 tins of dog food and 3 tins of cat food. Tins of dog food cost 54 p and tins of cat food cost 85 p. She gets $\frac{1}{4}$ off the total cost. She pays with a £20 note. How much change should she get?

$\frac{1}{5}$ of the pens in a box are blue. 40% of the pens are green. All the other pens are red. What fraction of the pens are red?

I had £100 given to me for my birthday. I saved $\frac{3}{5}$ of it and spent $\frac{1}{4}$ of it on CDs. Then I bought a magazine for £2.50. The rest of it I used for a present for my father.
i. How much was the present?
ii. What fraction was this of my original amount?

Not Given Total:

Two thirds of a number is 66. What's the number?

There are 15 girls. $\frac{3}{8}$ are boys. How many are boys?

Lara has some money. She spent £1.25 on a drink. She spent £1.60 on a sandwich. She has three-quarters of her money left. How much money did Lara have to start with?

On Saturday Lara read $\frac{2}{5}$ of her book. On Sunday she read the other 90 pages to finish the book. How many pages are there in Lara's book?

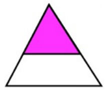
A bag contains $\frac{5}{8}$ lemons, $\frac{1}{6}$ pears and the rest are oranges. There are 15 oranges in the bag. How many pieces of fruit are there in the bag?

Klein reads 30 pages of a book on Monday and $\frac{1}{8}$ of the book on Tuesday. He completed the remaining $\frac{1}{4}$ of the book on Wednesday. How many pages are there in the book?

Bill and Cecily buy a packet of nuts. Bill eats a quarter of the nuts and Cecily eats a sixth of the nuts in the packet. What fraction of the number of nuts in the packet has been eaten? They give the remaining 63 nuts to Daniel. How many nuts were there in the full packet?

The butcher sells slices of turkey. Donald buys $\frac{3}{4}$ of the slices in the shop. Alisha then buys a sixth of the remaining slices. Alisha bought 3 slices. How many more slices did Alisha buy than Donald?

The diagram shows a shaded triangle inside a larger triangle. The area of the shaded triangle is $52\ cm^2$. The area of the shaded triangle is $\frac{4}{9}$ of the area of the larger triangle. Calculate the area of the larger triangle.



Mix Of All Types

Lucky Jim wins £1500 for being schoolmaster of the year. He spends $\frac{3}{5}$ of his sum taking his wife on holiday.
i. How much does he spend on the holiday?

Jim shares the rest of the winning equally between his 4 children.
ii. What fraction of the Jim's total winnings does each child receive?

Tom's cat drinks $\frac{3}{4}$ of a litre of milk each day
i. How many litres of milk will Tom's cat drink in 24 days?
ii. How long will 12 litres of milk last Tom's cat?

5 friends order a giant pizza. Sam eats $\frac{1}{4}$ of the pizza, Tim eats $\frac{1}{6}$ and Matt eats $\frac{1}{8}$ of it
i. What fraction of the pizza do Sam, Tim and Matt eat altogether?
ii. Ruth and Sarah share the rest of the pizza equally between them, what fraction of the pizza does Ruth eat?
iii. Aunty May uses $\frac{2}{5}$ of a ball of wool to knit 1 sock for Baby Bob. How many balls of wool does Auntie May need to knit 10 socks for Baby Bob?

5 children share a sum of money. Big brother takes $\frac{4}{9}$ of it and small sister takes $\frac{1}{3}$ of it.
i. What fraction of the sum of money remains?

The frightful four then share what is left over equally between them
ii. What fraction of the sum of money does each take?

A group of 5 friends eat 4 tortillas between them. Pablo eats $1\frac{1}{4}$ tortillas and Elena eats $\frac{7}{8}$ of a tortilla.
i. How many tortillas do Pablo and Elena eat altogether?
ii. How many tortillas remain uneaten?

The three other friends share equally what remains of the tortillas
iii. What fraction of a tortilla does each of these friends eat?

Sam is celebrating his birthday with 2 identical birthday cakes. Sam eats $\frac{1}{4}$ of a cake, his sister eats $\frac{1}{6}$ of a cake and his friends eat $1\frac{1}{3}$ cakes between them
i. How many cakes have been eaten altogether?
ii. Sam's mother and father share what is left of the cakes equally. What fraction of a cake does Sam's mother eat?
iii. $\frac{1}{3}$ of a class of 24 children are girls. $\frac{3}{4}$ of the girls and $\frac{1}{2}$ of the boys learn a musical instrument. How many children in the class learn a musical instrument

Challenges

Two thirds of a number is 3 more than three fifths of the number. What's the number?

Half of a number is 8 bigger than three sevenths of the number. What's the number?

Three quarters of a number is 36 less than the number. What's the number?

$\frac{1}{2}$ of a number is three times $\frac{1}{4}$ of 72. What is the number?

Jen's fish tank was filled half-way. She added 9 gallons of water to the talk and found that it was then 7/8 full. How much water can they tank hold altogether?